KATHOLIEKE UNIVERSITEIT LEUVEN

Faculty of Business and Economics

The Dynamics of Price Discovery in the Two-Tier Brussels Stock Exchange

Thi Ngoc Tuan Bui and Piet Sercu

DEPARTMENT OF ACCOUNTANCY, FINANCE AND INSURANCE (AFI)

AFI_0722

The Dynamics of Price Discovery in the Two-Tier Brussels Stock Exchange

11:39 on Friday 26$^{\text{th}}$ September, 2008

Abstract

In this paper, we investigate the dynamics of price discovery in the Brussels Stock Exchange for the spot and forward stock markets. Specifically, we quantitatively analyze each market's process in impounding new fundamental information about a stock's value into its market price. Similar to the permanent-transitory decomposition procedure put forward by Gonzalo and Ng (2001), we use a VECM (Vector Error Correction Model) and decompose the VECM residuals into the permanent and transitory innovations. However, we adjust their procedure to accommodate for the asynchronism problem in the Brussels forward and spot stock exchange. From the impulse response functions of the derived structural cointegration model after the decomposition, we apply the price discovery measure proposed by Yan and Zivot (2007), which is the absolute magnitude of cumulative price errors in the process of reflecting a one-unit change in the permanent innovation. In particular, we investigate which market makes less errors while incorporating the full one-unit increase in the permanent innovation into its price. Our finding is that the spot market outperforms the forwards one in the price discovery process. This result contradicts the price discovery role conventionally ascribed to the forward market.

JEL G14, G12, G1

The Dynamics of Price Discovery in the Two-Tier Brussels Stock Exchange

1 Introduction

This paper investigates the price discovery process in the Brussel Stock Exchange (BSE) during the period 1989 - 1996. In that period, the market still consisted of two trading tiers for stocks: a "spot" market tier with third-day delivery, and for the most active stocks a parallel "forward" tier with fixed-date delivery. Both segments were order-driven, and their opening prices—the only ones for which we have a fairly precise idea about the timing—were set via a call. The trading in the forward market started at 10 a.m, on the other hand, the opening spot prices were identified at 1.30 p.m. In addition, the forward market was cheaper, deeper, unhampered by price limits, convenient also for short sales, fully computerized, and played by the pros; so it looks likely to be the more efficient tier, with less noisy prices and doing most of the price discovery work. Surprisingly, though, using short-run price-discovery regressions similar to that of Margrabe and Silverman (1983), Bui & Sercu (2007) find evidence indicating the spot market may have been the more efficient one. The diagnosis is tentative, though, because their methodology is not robust to asynchronism problems in their data. In addition, their simple price-discovery models do not allow to discover the dynamics of how news gets incorporated into prices. In this paper we try to improve their analysis on both these counts. Specifically, this paper quantitatively examines whether the forward market is the first to fully reflect new fundamental information about a stock's value into its market price and whether it makes less errors during the process of fully impounding this information; and the methodology does not suffer from asynchronism problems between spot and forward prices.

This research falls within the literature on price discovery, which is one of the central functions of financial markets. It is generally defined as a process of asset's price adjustment to incorporate new information. For example, Schreiber and Schwartz (1986) interpret it as "the search for an equilibrium price", Baillie et. al. (2002) as gathering and interpreting news to determine financial asset's price, or Lehmann (2002) as "efficient and timely incorportation of the information implicit in investor trading into the market prices".

Within this price discovery literature, a two-step orthogonalization on the residuals of a Vector Error-Correction Model (VECM) of a cointegrated system proposed by Gonzalo & Ng

(2001) has been commonly used. In conventional VECM analysis, the residuals are usually viewed as innovations to the variables in the systems. However, these residuals do not reveal the source of the shocks. Gonzalo & Ng (2001) put forward a way of identifying the permanent and transitory shocks (P-T) from a system of cointegrated variables according to whether their effects are permanent or transitory. This isolation allows one to trace the propagating mechanism of each type of shocks. Applying their definition of innovations according to their degree of persistence, we adjust the VECM to accommodate for the asynchronism problem and suggest a slightly different and more direct solution to the task of decomposing the VECM residuals into permanent and transitory innovations.

After the decomposition into permanent and transitory components, the derived cointegrated system—commonly called a structural moving average (SMA) system—is used to perform impulse response analysis. Specifically, we examine the price adjustment process of the forward and spot markets following a one-unit change in the permanent innovation, i.e after the permanent innovation increases by BEF 1[1]. This analysis answers the question of how many days after this event it takes for each market to incorporate the full one-unit shock and whether the forward market is the quicker tier. In addition, applying the measure of price-discovery efficiency loss proposed by Yan and Zivot (2007), we estimate the accumulated error that each market makes while moving to the new fundamental value following the shock. A bootstrap method is used to estimate the confidence bands of the accumulated errors as well as the difference between these cumulative errors of the two markets. Our finding is that the forward market makes significantly more errors than the spot one, which again contradicts the superior role of the forward tier one would have expected.

The remainder of the paper is organized as follows. Section 2 describes the markets and the data. In section 3 we present the econometric framework, which includes the model of P-T decomposition, the solution to the model, and the measure of price discovery efficiency loss. The empirical results are reported in section 4. Section 5 includes.

[1]The belgian Frank (BEF) was the local currency in those days.

2 The Two-Tier Brussels Stock Exchange: Institutional Background

Brussels used to have not only its own stock market (the Brussel Stock Exchange (BSE), since 2001 integrated into Euronext), but even a two-tiered one: a "spot" market tier with third-day delivery, and for the most active stocks a parallel "forward" tier with fixed-date delivery. There used to be twenty-four fixed such settlement dates per year, implying that the trading periods typically lasted about two weeks—hence their name *quinzaine*, two-week period.[2] Details about the market organization are crucial for our analysis. In this section, we describe the price mechanisms in the forward and spot market and the delivery rules as they applied during the sample period.

2.1 The price mechanism in the forward tier

The forward market used to work via a pure public limit order book (which, during the sample period, was kept by a version of Toronto's Computer-Aided Trading System, CATS). Thus, although brokers were allowed to trade on their own account, they did not act as market makers, and their main role on the floor was to pass on the orders from the public to the exchange. At 9 p.m. the one-hour pre-market started, during which orders could be added or withdrawn and CATS displayed a continuously updated preliminary market-clearing price. Actual trading in the forward market started at 10 a.m., with a simultaneous call market for all stocks. That is, at 10 a.m. limit orders were matched as far as possible, and executed. For most stocks the opening represented a substantial part of the day's turnover. After the opening round, the interactive trading session or "continuous market" started (10:00-16:30). Throughout the continuous-market session, the four best unfilled limit orders on the buying and selling side were displayed on computer screens and could be taken up by any incoming new order. Only brokers saw the screens: at the time of the sample, individual investors just heard (or saw) the opening and close prices over the radio or on Teletext, at noon or in the afternoon. Orders could also be matched directly, between brokers or in-house, provided that the price was within the book's bid-ask spread and the trade was reported immediately to the exchange. Large trades, *i.e.* blocks of at least BEF 50m (EUR 1,250,000) could be

[2] The forward market has now disappeared, following an EU-directed "$T \leq t + 7$ days" rule implemented in the 1990s. London used to have a two-weekly fixed-delivery system too, Paris had delivery at the end of the month in its "forward" section for big stocks. (There also was a spot section for small stocks). Basel offered the choice between several delivery dates.

Table 1: **Tick Size in the Spot and Forward Market**

price range	price must be a multiple of	minimal percentage price change at lower end of scale	minimal percentage price change at top end of scale
BEF 1-500	1	100%	0.20%
BEF 502-1,500	2	0.40%	0.13%
BEF 1,505-5,000	5	0.33%	0.10%
BEF 5,010-10,000	10	0.20%	0.10%
BEF 10,025-50,000	25	0.25%	0.05%
BEF 50,050	50	0.10%	—

Key One BEF is approximately EUR 0.025.

crossed or traded outside the BSE (often in London or Paris), but had also to be reported immediately. There were no limits on consecutive forward price changes. Limit order and trade prices were rounded according to a shedule shown in Table 1. Until the 1996 reform, the exchange's minimum margin requirement for a forward trade was 25 percent, but the BSE left the enforcement of this rule to the individual brokers (who bore the default risk). Securities could be posted as margin; in fact, many investors left most or all of their stocks with a their broker—most shares are bearer securities—and used this portfolio as margin for forward positions. Thus, there was no opportunity cost associated with the margin.

Prices for all traded lots were shown, in sequence (but not time-stamped), in the official price list, later *De Tijd* and *L'Echo de la Bourse*. In the electronic records, only open/close/high/low are available.

2.2 The Spot Price Mechanism

Due to its lower volume, the spot market was fully computerized much later (in 1996). Like the forward tier, it was order-driven, but the implementation was fairly different. First, there was no pre-market, so that the opening price was much more subject to noise than the forward opening price even apart from volume effects. Second, because of the thinness of the market, for most stocks there was just one trading round per day. A continuous market existed only for the more active stocks (quoted on the *"corbeille"* segment), and even this continuous market was not very active. Third, there was no centralized public order book kept by the exchange. Rather, a few specialist brokers each kept their own books, and met sometime between 1 and 1.30 p.m. on the Exchange's floor to aggregate their information and identify the price that maximizes trade from the combined order book. Fourth, for stocks that were not traded on the parallel forward market there were daily price limits of 5 percent (for very thinly traded stocks, traded on the *parket* segment) or 10 percent (for other stocks, traded on the "*corbeille*" market); and, in the *corbeille* market, subsequent intraday price changes could not exceed 5 percent.

The actual pricing and trading was organized by a BSE official, who started by crying out a price proposal. This price proposal equaled the price that maximized trade from the order book if that price was within the price change limits; if not, the official announced the price limit itself. In addition to the price proposal, the official also announced the direction of the imbalance. If there was an excess supply (demand) at the proposed price, additional purchase (sale) orders from the floor were sollicited to reduce the imbalance in the book. If the remaining imbalance between supply and demand at the price limit was less than 50 percent, the specialist would decide to 'reduce' most or all orders on the excess side, *i.e.* execute only part of each order; the transaction price was then published in the financial press with the qualification *"sellers reduced"* or *"buyers reduced"*. If at the price limit the imbalance between supply and demand remained huge even after solliciting orders from the floor, there was no trade at all, and the price limit was published as an indicative price. In practice, however, when the imbalance was only slightly larger than 50 percent the stock's specialist brokers often added buy or sell orders for their own account to prevent no-trade (and no-income) days.

As, around 1990, the spot market list contained about 300 stocks, the stock-by-stock opening-call prices were set more or less sequentially; the exact timing of each stock's spot fixing was not registered.

The spot market had two sub-tiers. For about half the stocks, there was just one daily fixing; this was called the *parket* market. For stocks quoted on the *corbeille*, the fixing was followed by the traditional (blackboard-and-chalk) version of the continuous market: unfilled orders were chalked onto the blackboard and could be picked up from the floor, and orders could also be matched directly on the floor at a price within the book's spread. For the *corbeille* market, prices for all traded lots were shown, in sequence (but not time-stamped), in the official price list but in the electronic records, only open/close/high/low are available. For the *parket* stocks there is just the single price.

2.3 Settlement Rules

For the BSE, the other details of the actual settlement were similar for both market tiers. The buyer payed via a bank transfer rather than by check. This means that there was no "mail

Table 2: **Transaction costs, spot and forward, 1990**

item	cost of spot trades	cost of forward trades
BSE Commission	0.03%, max BEF 6 000†	
Transaction Tax	0.17%, max BEF 10 000	
Brokerage fees		
- fixed part	BEF 200*	
- variable part:		
order BEF 1-5m	1%	.8%
order BEF 5-10m	.8%	.6%
order BEF 10-20m	.4%	.3%
order BEF 20-30m	$\geq$ BEF 130 000‡	.2%
order $\geq$ BEF 30m	$\geq$ BEF 130 000‡	$\geq$ BEF 120 000‡

† : 40 BEF is worth approx. 1 EUR; * : plus BEF 100 for the buyer if physical delivery is asked; ‡ : negotiable, with the stated amounts as minima. Thus, a smallish trade of BEF 250,000 (approx. EUR 6.250) would cost 1.29 percent spot, and 1.09 percent forward. For an order of BEF 30m, the cost difference may be as small as 10,000/30,000,000 = .033 percent.

float" on the payment side. Still, the value dates for buyer and seller did not match perfectly: the buyer's value date is one day before the actual settlement day, and the seller obtains value one day after settlement.

Delivery of the stock could mean actual physical delivery of the piece of paper, if the buyer desired so. Alternatively, the buyer could ask that his or her purchase be recorded with a netting and depository institution, the *Caisse Interprofessionelle/Interprofessionale Kas* (CIK). The CIK merely netted the physical deliveries across brokers if actual delivery is asked, and held the paper on behalf of investors that did not demand physical delivery. Thus, the CIK was not a clearing house in the usual sense: it did not act as a central counterpart, nor did it cancel an individual investor's earlier purchases against subsequent sales (or *vice versa*) within one settlement period. There was some informal clearing by brokers, though: brokers did not exact delivery and payment for a forward transaction that was reversed later on via the same brokerage house and within the same quinzaine.

One function of the forward market, therefore, was to reduce the cost and hassle of mutually offsetting stock deliveries and payments for trades that had been closed out within the same quinzaine. This partly explains why, unlike in currency markets, in the forward tier the transaction costs for small trades were somewhat lower than in the spot tier (as illustrated in Table 2).[3] A second useful feature of the forward tier is that it allows one to take short positions until the end of the quinzaine, positions that could then be rolled over fairly easily. In Belgium, there was no formal legal framework for asset borrowing and spot short-selling until the 1991 Stock Market Reform Act, so until then the forward market provided the sole organized opportunity for short positions. A third function of the forward market was to provide the equivalent of buying on margin: the actual payment was deferred until the end of the *quinzaine* (at which moment the forward contract could be rolled over), and the buyer just posted the 25 percent security. Since leveraged buying was possible in the forward market, no organized system of buying on margin was set up in the spot market.

2.4 Possible clientele and differential information aspects

It is fair to say that the organization of the forward markets was superior: it was fully computerized by the late 80s, had a pre-market, enjoyed lower costs and no price limits, and was much deeper. In addition (or, perhaps, as a result of the above), conventional wisdom within the financial community held that there also was an clientele- and efficiency-related form of segmentation.[4] Indeed, because of its shorting facilities and the absence of price limits, the forward market had a somewhat more speculative reputation, to the extent that conservative firms (such as the major banks) have long resisted a forward listing. Because of this speculative image, the forward market was considered to be the market for the more professional agents, while less sophisticated investors were said to prefer the spot market. Having no systematic and fast access to news during working hours, these amateur traders allegedly reacted slower than the professionals. In the terminology of Garbade and Silber (1983), this view hypothesizes that the forward market was the price discoverer, while the spot market was just a (lagging) satellite market. This hypothesis is the central issue of the paper.

2.5 Taxes

A last relevant detail is income tax. For brokers or corporations, all interest received or paid and all short-term capital gains or losses are fully taxable or deductible. So if brokers or corporations dominate the market in the sense that they are systematically the marginal

[3] Another reason for the lower transaction costs might have been the fact that the forward market tended to have larger volumes than the spot market for the same stock.

[4] We are indebted to the late Prof. Emeritus Van Essche for this suggestion.

traders, taxes are neutral. Under personal taxation rules, capital gains or losses are not taxable or deductible; nor can one deduct interest costs incurred to finance short-term trades; and interest income is *de facto* taxed at the withholding tax (10 percent at the time). In short, also for private persons the gross rate is the relevant interest rate, unless the marginal traders are buyers of stock for whom the opportunity cost is a deposit.

Dividends are largely tax-exempt for corporations; for individuals, a 25-percent withholding tax applies. Unpublished tests show that the average price drop on ex-dividend day was equal to the dividend net of the withholding tax—20 percent before 1984, and 25 percent thereafter. All returns are accordingly computed from dividends net of withholding taxes.

We conclude the descriptive section with some information on the data.

2.6 Data Description

The sample period starts early 1989, at which time the forward markets was fully computerized, and ends in 1996; in 1997 the forward market disappeared. Euronext's historic-data CDs for that period include the opening spot price per day, and, for the forward market, the daily opening, high and low, and close price. Data on dividends, bonus dividends, splits, and rights issues[5] were missing, and were hand-collected from *Memento der Effecten*, a trade publication, and from *De Tijd*, which published the Dutch-language version of the Official Price List.

We discarded foreign stocks, about half of the list, since price discovery for these shares probably comes from abroad anyway. So we started from data on 119 Belgian stocks traded on both the spot and forward tiers of the Brussels Stock Exchange during the period 1989-1996. Some data cleaning was required: 17 stocks are excluded due to an insufficient number of observations (too many missing data points), 31 stocks are connected to the others due to the change in the name or code after stock split or merger. Accordingly, 71 stocks remain. All unusually large forward premia or large changes in the prices were double-checked with the prices posted on the hard copies of *De Tijd*, including the next-day rectifications for typos. All prices that are qualified as sellers reduced, buyers reduced, or indicative, were considered to be missing observations.[6] The raw prices were then corrected for differences in time to

[5]The subscription right is represented by a coupon and can be traded separately the moment the stock goes ex this coupon. The market values of these "scripts" are very noisy so we worked with the standard theoretical value of a subscription right.

[6]Since limit-order prices are from a grid, the supply and demand schedules do rarely "cross" precisely: there almost always is some excess supply or demand left. Whenever the market holder could not close the gap by

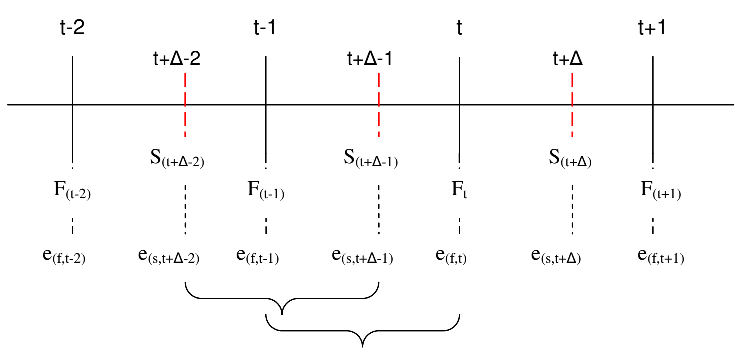
settlement, by discounting them for the appropriate number of days. We used the one-week call-money rate and the [calendar days]/360 time convention that then applied outside the interbank market for BEF. Thus, both price series can be viewed as separate but comparable assessments of a true spot value. The impact of the discounting turned out to be so minute that it could have been omitted without affecting any conclusion.

3 Econometric Framework

3.1 Preliminaries

Let $X_t = (F_t \; S_{t+\triangle})'$ denote a 2×1 vector of prices for the stock trading in the forward and spot markets respectively (F_t is the opening forward price observed at 10 a.m. on day t and $S_{t+\triangle}$ is the opening spot price observed at 1.30 p.m. on day t). Figure (3) below represents the asynchronism between the two markets.

Figure 1: **Time Asynchronism**



We assume each of these prices contains a random walk component so that X_t is a nonstationary process. These prices are integrated of order 1 or $I(1)$, and the price changes, $\triangle X_t$, are integrated of order zero, or $I(0)$. We assume that $\triangle X_t$ has a bivariate Vector Moving Average

solliciting additional orders from the floor but the imbalance is small, all orders were executed for the same proportion ("buyers (or sellers) reduced"). Whenever the imbalance was large, no trade was allowed and an indicative price was published.

(VMA) or Wold presentation below:

$$\triangle X_t = \Psi(L)e_t = e_t + \Psi_1 e_{t-1} + \Psi_2 e_{t-2} + ..., \tag{1}$$

$$\Psi(L) = \sum_{k=0}^{\infty} \Psi_k L^k, \Psi_0 = I_2, \tag{2}$$

where $e_t = (e_{f,t}\ e_{s,t+\triangle})^{'}$ is a 2×1 vector of innovations satisfying $\mathrm{E}(e_t) = 0$ and

$$\mathrm{E}[e_t e_s^{'}] = \begin{cases} \mathrm{E}\left[\begin{pmatrix} e_{f,t} \\ e_{s,t+\triangle} \end{pmatrix} \begin{pmatrix} e_{f,t} & e_{s,t+\triangle} \end{pmatrix}\right] = \begin{pmatrix} \mathrm{var}(e_{f,t}) & \mathrm{cov}(e_{f,t}, e_{s,t+\triangle}) \\ \mathrm{cov}(e_{f,t}, e_{s,t+\triangle}) & \mathrm{var}(e_{s,t+\triangle}) \end{pmatrix} \neq 0 & \text{if } t = s, \\ \mathrm{E}\left[\begin{pmatrix} e_{f,t} \\ e_{s,t+\triangle} \end{pmatrix} \begin{pmatrix} e_{f,t-1} & e_{s,t+\triangle-1} \end{pmatrix}\right] = \begin{pmatrix} 0 & \mathrm{cov}(e_{f,t}, e_{s,t+\triangle-1}) \\ 0 & 0 \end{pmatrix} \neq 0 & \text{if } t = s-1, \\ 0 & \text{otherwise.} \end{cases} \tag{3}$$

The variance-covariance matrix $\mathrm{E}[e_t e_{t-1}^{'}]$ is non-zero because of the time-overlap between the two periods $(t-1, t)$ and $(t+\triangle-2, t+\triangle-1)$ (see Figure 3). In case of time-synchronism, that is when $\triangle = 0$, this variance-covariance matrix is zero. So, from the VMA representation (1), the only adjustment of the standard VMA due to the presence of the asynchronism $\triangle$ between F_t (and $S_{t+\triangle}$ and thus between $e_{f,t}$ and $e_{s,t+\triangle}$) is the non-zero variance-covariance matrix $\mathrm{E}[e_t e_{t-1}^{'}]$.

The matrix polynomial $\Psi(L) = \Psi(1) + (1-L)\Psi^*(L)$ has the property that $\Psi(z)$ is 1-summable, and $\Psi^*(z)$ is full rank everywhere on $|z| \leq 1$.

The Beveridge-Nelson (BN) decomposition of $\Psi(L)$ results in the representation of the price levels:

$$X_t = X_0 + \Psi(1)\sum_{s=1}^{t} e_s + \Psi^*(L)e_t \tag{4}$$

In this presentation, $\Psi(1)$ contains the long-run impact of a innovation on each of the prices. Hasbrouck (1995) shows that since the difference between the two prices is stationary, this cointegration system has a cointegration vector $\beta' = (1\ \ -1)$. Consequently, $\beta'\Psi(1) = 0$ and the rows of $\Psi(1)$ are identical. Let $\Psi(1) = \begin{pmatrix} \psi_1 & \psi_2 \\ \psi_1 & \psi_2 \end{pmatrix}$.

In order to estimate the matrix polynomial $\Psi(L)$ of the VMA difference presentation (1), we first regress a VECM representation of X_t and then deduce the elements of $\Psi(L)$ in the same way as Yan & Zivot (2007) do. A VECM representation of finite order of X_t is approximated by:

$$\triangle X_t = \gamma\alpha^{'} X_{t-1} + \Gamma(L)\triangle X_{t-1} + e_t, \tag{5}$$

where $\Gamma(L)$ is of finite order $(K-1)$.

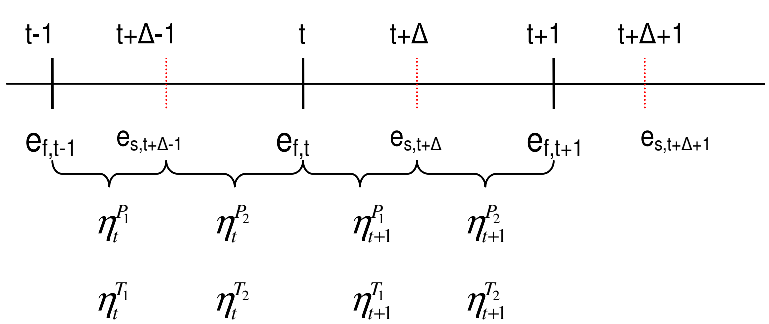
3.2 Model: P-T Decomposition & Transformation from VMA to SMA representation

A clear interpretation of price discovery is possible in a structural model, in which the types of shocks are identified. The standard model, first proposed by Gonzalo & Ng (2001), allows for a modest departure from the homogenous-expectations setting, in the sense that the two market tiers receive the same signals but may weigh them differently. One signal is the true fundamental news that triggers a change in the economic value of the asset, and the other signal just reflects friction or noise. Almost tautologically, the effect of true news on the value generates a permanent change, while the noise signal is transitory, i.e., undone in the end. Denoting these signals by η^P and η^T, respectively, and still in the absence of an asynchrony problem, the standard model then relates the innovations in the spot and forward prices to these signals as follows:

$$\begin{cases} e_{f,t} = d_1\, \eta_t^P + d_2\, \eta_t^T\,, \\ e_{s,t} = d_3\, \eta_t^P + d_4\, \eta_t^T\,. \end{cases} \tag{6}$$

We need to modify the notation and the model to accommodate for the asynchronism problem between the two markets. Divide the one-day period $[t-1, t)$ into two subperiods $[t-1, t+\triangle-1)$ and $[t+\triangle-1, t)$ as illustrated in figure (2)—basically the morning and the afternoon/overnight period, respectively. Let the permanent and transitory shocks in the subperiod $[t-1, t+\triangle-1)$ be $\eta_{1,t} = (\eta_t^{P_1}\ \eta_t^{T_1})'$ and in the subperiod $[t+\triangle-1, t)$ be $\eta_{2,t} = (\eta_t^{P_2}\ \eta_t^{T_2})'$. Those shocks then underly the innovations in the VEC model with the proviso that in the spot market the trading day starts at noon rather than in the morning:

Figure 2: **Permanent-Transitory Decomposition**



$$\begin{cases} e_{f,t} = d_1(\eta_t^{P_1} + \eta_t^{P_2}) + d_2(\eta_t^{T_1} + \eta_t^{T_2}), \\ e_{s,t+\triangle} = d_3(\eta_t^{P_2} + \eta_{t+1}^{P_1}) + d_4(\eta_t^{T_2} + \eta_{t+1}^{T_1}). \end{cases}$$

or

$$\begin{pmatrix} e_{f,t} \\ e_{s,t+\triangle} \end{pmatrix} = \begin{pmatrix} d_1 & d_1 & 0 & d_2 & d_2 & 0 \\ 0 & d_3 & d_3 & 0 & d_4 & d_4 \end{pmatrix} \begin{pmatrix} \eta_t^{P_1} \\ \eta_t^{P_2} \\ \eta_{t+1}^{P_1} \\ \eta_t^{T_1} \\ \eta_t^{T_2} \\ \eta_{t+1}^{T_1} \end{pmatrix}, \tag{7}$$

where $D_0 = \begin{pmatrix} d_1 & d_1 & 0 & d_2 & d_2 & 0 \\ 0 & d_3 & d_3 & 0 & d_4 & d_4 \end{pmatrix}$ contains the initial impacts of the structural shocks on $\triangle X_t$, and where the structural shocks are assumed to be serially and mutually uncorrelated.

In the original model with synchronized markets, the two innovations allow us to back out the two signals once the four weights d_i have been identified. (Actually, one of the ds can be fixed arbitrarily since its effect can be captured by rescaling the etas.) With asynchronous prices, however, we have four etas underlying the two innovations, so at best we can obtain the second moments of the variables, not the values themselves. To identify these moments and the weights d we also need to impose one more condition. In a first approach we assume that the relative volatility of permanent and transitory shocks is the same in the two sub-periods:

$$\frac{\text{var}(\eta_t^{P_1})}{\text{var}(\eta_t^{P_2})} = \frac{\text{var}(\eta_t^{T_1})}{\text{var}(\eta_t^{T_2})} = \rho. \tag{8}$$

This ratio of volatilities can be estimated from the observed residuals e_t as follows:

$$\text{cov}(e_{f,t}, e_{s,t+\triangle}) = d_1 d_3 \text{var}(\eta_t^{P_2}) + d_2 d_4 \text{var}(\eta_t^{T_2}), \tag{9}$$

$$\text{cov}(e_{f,t}, e_{s,t+\triangle-1}) = d_1 d_3 \text{var}(\eta_t^{P_1}) + d_2 d_4 \text{var}(\eta_t^{T_1}). \tag{10}$$

Replacing $\text{var}(\eta_t^{P_1})$ by $\rho\text{var}(\eta_t^{P_2})$ and $\text{var}(\eta_t^{T_1})$ by $\rho\text{var}(\eta_t^{T_2})$, from equation (8) we can then estimate ρ:

$$\rho = \frac{\text{cov}(e_{f,t}, e_{s,t+\triangle-1})}{\text{cov}(e_{f,t}, e_{s,t+\triangle})}. \tag{11}$$

If the P-T decomposition in (7) is identified, i.e. coefficients d_1, d_2, d_3, and d_4 are estimated, we can trace the propagating mechanism of the permanent and transitory shocks on the prices. Therefore, the task is to find a solution for d_1, d_2, d_3, and d_4 so that the a priori restrictions imposed on the permanent and transitory the shocks are satisfied. We first rewrite the P-T decomposition in (7) in the following way:

$$\begin{pmatrix} e_{f,t} \\ e_{s,t+\triangle} \end{pmatrix} = \begin{pmatrix} d_1 & d_1 & d_2 & d_2 \\ d_3 L^{-1} & d_3 & d_4 L^{-1} & d_4 \end{pmatrix} \begin{pmatrix} \eta_t^{P_1} \\ \eta_t^{P_2} \\ \eta_t^{T_1} \\ \eta_t^{T_2} \end{pmatrix}, \tag{12}$$

$$= G(L)\,\eta_t, \tag{13}$$

where $G(L) = \begin{pmatrix} d_1 & d_1 & d_2 & d_2 \\ d_3 L^{-1} & d_3 & d_4 L^{-1} & d_4 \end{pmatrix}$, (with L^{-1} being a lead operation, e.g. $L^{-1}e_t = e_{t+1}$), and $\eta_t = \begin{pmatrix} \eta_t^{P_1} \\ \eta_t^{P_2} \\ \eta_t^{T_1} \\ \eta_t^{T_2} \end{pmatrix}$.

With a decomposition of this form, the VMA presentation (1) becomes a structural moving average (SMA). Let us define

$$D(L) := \Psi(L)\,G(L), \tag{14}$$

whose eight elements will be denoted as $D(L) = \begin{pmatrix} d_{11}(L) & d_{12}(L) & d_{13}(L) & d_{14}(L) \\ d_{21}(L) & d_{22}(L) & d_{23}(L) & d_{24}(L) \end{pmatrix}$. Then the SMA is obtained as

$$\begin{pmatrix} \triangle F_t \\ \triangle S_{t+\triangle} \end{pmatrix} = \Psi(L)\,G(L)\,\eta_t, \tag{15}$$

$$= D(L)\,\eta_t, \tag{16}$$

$$= \begin{pmatrix} d_{11}(L) & d_{12}(L) & d_{13}(L) & d_{14}(L) \\ d_{21}(L) & d_{22}(L) & d_{23}(L) & d_{24}(L) \end{pmatrix} \begin{pmatrix} \eta_t^{P_1} \\ \eta_t^{P_2} \\ \eta_t^{T_1} \\ \eta_t^{T_2} \end{pmatrix}, \tag{17}$$

with $d_{ij}(L)$ (for $i = 1$, 2 and $j = 1$ to 4) being scalar polynomials:

$$d_{ij}(L) = d_{ij,0} + d_{ij,1}L + d_{ij,2}L^2 + \ldots. \text{ with } i = 1 \text{ or } 2 \text{ and } j = 1 \text{ to } 4. \tag{18}$$

Recall that the permanent innovations $\eta_t^{P_1}$ and $\eta_t^{P_2}$ are interpreted as new information on the fundamental value of the underlying asset, released during the corresponding periods $[t, t + \triangle - 1)$ and $[t + \triangle - 1, t)$. Being fundamental, this information permanently moves the market prices. The defining characteristic of $\eta_t^{P_1}$ and $\eta_t^{P_2}$ therefore is that they each have a one-to-one long-run effect on the price levels in each market tier. So we have the two conditions

$$\lim_{k\to\infty} \frac{\partial \mathrm{E}_t[F_{t+k}]}{\partial \eta_t^{P_1}} = \lim_{k\to\infty} \sum_{l=0}^{k} \frac{\partial \mathrm{E}_t[\triangle F_{t+l}]}{\partial \eta_t^{P_1}} = d_{11}(1) = 1, \tag{19}$$

and

$$\lim_{k\to\infty} \frac{\partial \mathrm{E}_t[S_{t+\triangle+k}]}{\partial \eta_t^{P_1}} = \lim_{k\to\infty} \sum_{l=0}^{k} \frac{\partial \mathrm{E}_t[\triangle S_{t+\triangle+l}]}{\partial \eta_t^{P_1}} = d_{21}(1) = 1, \tag{20}$$

for the morning news. Similarly, for the afternoon/overnight news we need

$$\lim_{k\to\infty} \frac{\partial \mathrm{E}_t[F_{t+k}]}{\partial \eta_t^{P_2}} = \lim_{k\to\infty} \sum_{l=0}^{k} \frac{\partial \mathrm{E}_t[\triangle F_{t+l}]}{\partial \eta_t^{P_2}} = d_{12}(1) = 1, \tag{21}$$

and

$$\lim_{k\to\infty} \frac{\partial \mathrm{E}_t[S_{t+\triangle+k}]}{\partial \eta_t^{P_2}} = \lim_{k\to\infty} \sum_{l=0}^{k} \frac{\partial \mathrm{E}_t[\triangle S_{t+\triangle+l}]}{\partial \eta_t^{P_2}} = d_{22}(1) = 1. \tag{22}$$

The transitory innovation η_t^T, in contrast, is the result of all non information-related frictions, such as the trading by uninformed or liquidity traders, inventory adjustments, and any random temporary order imbalance. The defining characteristic of η_t^T is that it has no long-run effect on the price levels:

$$\lim_{k\to\infty} \frac{\partial \mathrm{E}_t(F_{t+k})}{\partial \eta_t^{T_1}} = \lim_{k\to\infty} \sum_{l=0}^{h} \frac{\partial \mathrm{E}_t(\Delta F_{t+l})}{\partial \eta_t^{T_1}} = d_{13}(1) = 0. \tag{23}$$

$$\lim_{k\to\infty} \frac{\partial \mathrm{E}_t(S_{t+\triangle+k})}{\partial \eta_t^{T_1}} = d_{23}(1) = 0. \tag{24}$$

$$\lim_{k\to\infty} \frac{\partial \mathrm{E}_t(F_{t+k})}{\partial \eta_t^{T_2}} = d_{14}(1) = 0. \tag{25}$$

$$\lim_{k\to\infty} \frac{\partial \mathrm{E}_t(S_{t+\triangle+k})}{\partial \eta_t^{T_2}} = d_{24}(1) = 0. \tag{26}$$

As a corollary, we infer from (19)-(26) that

$$\begin{aligned} D(1) &= \begin{pmatrix} d_{11}(1) & d_{12}(1) & d_{13}(1) & d_{14}(1) \\ d_{21}(1) & d_{22}(1) & d_{23}(1) & d_{24}(1) \end{pmatrix}, && (27) \\ &= \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{pmatrix}. && (28) \end{aligned}$$

Moreover, since $D(L) \equiv \Psi(L)\, G(L)$, we have

$$\begin{aligned} D(1) &= \Psi(1)\, G(1), && (29) \\ &= \begin{pmatrix} \psi_1 & \psi_2 \\ \psi_1 & \psi_2 \end{pmatrix} \begin{pmatrix} d_1 & d_1 & d_2 & d_2 \\ d_3 & d_3 & d_4 & d_4 \end{pmatrix}, && (30) \\ &= \begin{pmatrix} \psi_1 d_1 + \psi_2 d_3 & \psi_1 d_1 + \psi_2 d_3 & \psi_1 d_2 + \psi_2 d_4 & \psi_1 d_2 + \psi_2 d_4 \\ \psi_1 d_1 + \psi_2 d_3 & \psi_1 d_1 + \psi_2 d_3 & \psi_1 d_2 + \psi_2 d_4 & \psi_1 d_2 + \psi_2 d_4 \end{pmatrix}. && (31) \end{aligned}$$

Combining (28)and (31) we obtain the following conditions for $G(L)$:

$$\psi_1 d_1 + \psi_2 d_3 = 1, \tag{32}$$

$$\psi_1 d_2 + \psi_2 d_4 = 0. \tag{33}$$

So the transformation from the VMA in (1) to the SMA in (17) is complete if the coefficients of the matrix $G(L)$ (first mentioned in (13)) are identified, satisfying the conditions in (32) and (33). In such a case, $D(L)$ is identified as in (14).

3.3 Solving the model

Restate the model. Starting from VMA: $\triangle X_t = \Psi(L)e_t$, the task is to find a solution for coefficients d_1, d_2, d_3, and d_4 such that the following three conditions are met:

(i) P-T decomposition (by definition) $\begin{cases} e_{f,t} = d_1(\eta_t^{P_1} + \eta_t^{P_2}) + d_2(\eta_t^{T_1} + \eta_t^{T_2}), \\ e_{s,t+\triangle} = d_3(\eta_{t+1}^{P_1} + \eta_t^{P_2}) + d_4(\eta_{t+1}^{T_1} + \eta_t^{T_2}). \end{cases}$

(ii) Defining characteristics of P-T innovations (Conditions (32) and (33)):

$$\psi_1 d_1 + \psi_2 d_3 = 1, \tag{34}$$
$$\frac{d_2}{d_4} = -\frac{\psi_2}{\psi_1}. \tag{35}$$

(iii) Orthogonality condition: by definition,

$$\mathrm{cov}(\eta_t^{P_i}, \eta_t^{T_j}) = 0 \text{ with } i,\ j = 1 \text{ or } 2. \tag{36}$$

(iv) Zero serial correlation:

$$\mathrm{cov}(\eta_t^{P_1}, \eta_t^{P_2}) = 0, \tag{37}$$
$$\mathrm{cov}(\eta_t^{T_1}, \eta_t^{T_2}) = 0. \tag{38}$$

(v) Constant noise/signal variance ratios:

$$\mathrm{var}(\eta_t^{P_1}) = \rho\mathrm{var}(\eta_t^{P_2}), \tag{39}$$
$$\mathrm{var}(\eta_t^{T_1}) = \rho\mathrm{var}(\eta_t^{T_2}). \tag{40}$$

These five conditions suffice to uniquely solve for the four coefficients d_1, d_2, d_3, and d_4, as follows. From conditions (i) and (iv), we have

$$\mathrm{var}(e_{f,t}) = d_1^2(\mathrm{var}(\eta_t^{P_1}) + \mathrm{var}(\eta_t^{P_2})) + d_2^2(\mathrm{var}(\eta_t^{T_1}) + \mathrm{var}(\eta_t^{T_2})), \tag{41}$$
$$\mathrm{var}(e_{s,t+\triangle}) = d_3^2(\mathrm{var}(\eta_{t+1}^{P_1}) + \mathrm{var}(\eta_t^{P_2})) + d_4^2(\mathrm{var}(\eta_{t+1}^{T_1}) + \mathrm{var}(\eta_t^{T_2})), \tag{42}$$
$$\mathrm{cov}(e_{f,t}, e_{s,t+\triangle}) = d_1 d_3 \mathrm{var}(\eta_t^{P_2}) + d_2 d_4 \mathrm{var}(\eta_t^{T_2}), \tag{43}$$
$$\mathrm{cov}(e_{f,t}, e_{s,t+\triangle-1}) = d_1 d_3 \mathrm{var}(\eta_t^{P_1}) + d_2 d_4 \mathrm{var}(\eta_t^{T_1}). \tag{44}$$

Replacing $\mathrm{var}(\eta_t^{P_1})$ by $\rho\mathrm{var}(\eta_t^{P_2})$ and $\mathrm{var}(\eta_t^{T_1})$ by $\rho\mathrm{var}(\eta_t^{T_2})$, the above four equations become:

$$\begin{aligned}
\mathrm{var}(e_{f,t}) &= d_1^2(1+\rho)\mathrm{var}(\eta_t^{P_2}) + d_2^2(1+\rho)\mathrm{var}(\eta_t^{T_2}), && (45)\\
\mathrm{var}(e_{s,t+\triangle}) &= d_3^2(1+\rho)\mathrm{var}(\eta_t^{P_2}) + d_4^2(1+\rho)\mathrm{var}(\eta_t^{T_2}), && (46)\\
\mathrm{cov}(e_{f,t}, e_{s,t+\triangle}) &= d_1 d_3\mathrm{var}(\eta_t^{P_2}) + d_2 d_4\mathrm{var}(\eta_t^{T_2}), && (47)\\
\rho &= \frac{\mathrm{cov}(e_{f,t}, e_{s,t+\triangle-1})}{\mathrm{cov}(e_{f,t}, e_{s,t+\triangle})}. && (48)
\end{aligned}$$

The four equations (34), (45), (46), and (47) are sufficient to solve for the four unknowns $d_1, d_3, \mathrm{var}(\eta_t^{P_2})$ and $\mathrm{var}(\eta_t^{T_2})$ as followings (see the appendix for the details):

$$\begin{aligned}
d_1 &= \frac{(1+\rho)\psi_2\mathrm{cov}(e_{f,t}, e_{s,t+\triangle}) + \psi_1\mathrm{var}(e_{f,t})}{\psi_1^2\mathrm{var}(e_{f,t}) + 2(1+\rho)\psi_1\psi_2\mathrm{cov}(e_{f,t}, e_{s,t+\triangle}) + \psi_2^2\mathrm{var}(e_{s,t+\triangle})}, && (49)\\
d_3 &= \frac{(1+\rho)\psi_1\mathrm{cov}(e_{f,t}, e_{s,t+\triangle}) + \psi_2\mathrm{var}(e_{s,t+\triangle})}{\psi_1^2\mathrm{var}(e_{f,t}) + 2(1+\rho)\psi_1\psi_2\mathrm{cov}(e_{f,t}, e_{s,t+\triangle}) + \psi_2^2\mathrm{var}(e_{s,t+\triangle})}. && (50)
\end{aligned}$$

Since one d can be chosen arbitrarily, we fix the two remaining weights such that they meet (35):

$$\begin{cases} d_2 = -\psi_2, \\ d_4 = \psi_1. \end{cases} \tag{51}$$

Equations (49), (50), and (51) are the complete solution for the matrix $G(L)$. Therefore, the permanent and transitory decomposition has been identified in the sense that the weights underlying the spot and forward innovations are known.

3.4 Measuring Price Discovery

Once the coefficient matrix $G(L)$ has been solved as presented above, the SMA presentation is identified too:

$$\begin{pmatrix} \triangle F_t \\ \triangle S_{t+\triangle} \end{pmatrix} = \underset{2\times 4}{D(L)} \underset{4\times 1}{\eta_t} = \begin{pmatrix} d_{11}(L) & d_{12}(L) & d_{13}(L) & d_{14}(L) \\ d_{21}(L) & d_{22}(L) & d_{23}(L) & d_{24}(L) \end{pmatrix} \begin{pmatrix} \eta_t^{P_1} \\ \eta_t^{P_2} \\ \eta_t^{T_1} \\ \eta_t^{T_2} \end{pmatrix}, \tag{52}$$

where the polynomial $D(L)$ can be identified via equation (14).

From this SMA presentation, we investigate the relative speed of price adjustment toward the stock's new equilibrium price following a one-unit change in the permanent innovation η_t^P. More specifically, we estimate how many days after the event it takes before each market has fully incorporated the one-unit change in the shock into its price. The quicker market has a greater contribution to price discovery.

From (52), the expected price response in each market, k days after a one-unit increase, to the permanent shock η_t^P is

$$f_{i,k}^F = \frac{\partial \mathrm{E}_t[F_{t+k}]}{\partial \eta_t^{P_i}} = \sum_{l=0}^{k} \frac{\partial \mathrm{E}_t[\triangle F_{t+l}]}{\partial \eta_t^{P_i}} = \sum_{l=0}^{k} d_{1i,l},\ k = 0,\ 1,\ ...;\ i = 1 \text{ or } 2 \tag{53}$$

$$f_{i,k}^S = \frac{\partial \mathrm{E}_t[S_{t+\triangle+k}]}{\partial \eta_t^{P_i}} = \sum_{l=0}^{k} \frac{\partial \mathrm{E}_t[\triangle S_{i,t+\triangle+l}]}{\partial \eta_t^{P_i}} = \sum_{l=0}^{k} d_{2i,l},\ k = 0,\ 1,\ ...;\ i = 1 \text{ or } 2 \tag{54}$$

From this response function, we estimate for each stock the price adjustment speeds. These parameters, k_f and k_s, are defined as the smallest number of days needed to ensure the price response $f_{i,k_f}^F = 1$ and $f_{i,k_s}^S = 1$.

As a second measure we adopt the price-discovery efficiency loss of Yan & Zivot (2007). The dynamics efficiency of market i at a given horizon k in response to a one-unit permanent shock may be characterized by $f_{i,k} - 1$, that is, the difference between expected price response, $f_{i,k}$, and the long-run response, one unit. Given a non-negative loss function L, Yan and Zivot define the price discovery efficiency loss (PDEL) for market i as the accumulated efficiency loss:

$$PDEL_i^F(K^*) = \sum_{k=0}^{K^*} |f_{i,k}^F - 1| \text{ with } i = 1,\ 2, \tag{55}$$

where K^* is a truncation lag chosen such that $f_{i,K^*}^F \approx 1$. For the spot market, similarly,

$$PDEL_i^S(K^*) = \sum_{k=0}^{K^*} |f_{i,k}^S - 1| \text{ with } i = 1,\ 2, \tag{56}$$

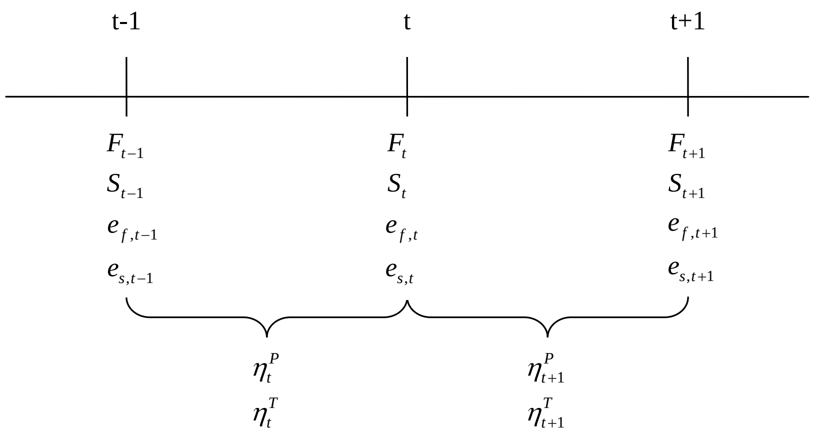
where K^* is a truncation lag chosen such that $f_{i,K^*}^S \approx 1$.

3.5 Time Synchronism and Comparison with P-T Decomposition of Gonzalo & Ng (2001)

When $\triangle = 0$, the model collapses to the case of synchronous prices (the prices F_t and S_t are observed at the same moment on day t), as pictured in Figure 3. In this case, we also decompose the standard VECM residuals $e_{f,t}$ and $e_{s,t}$ into P-T innovations η_t^P and η_t^T. Starting from the asynchronous case, we can set $\eta_t^{P_1} = \eta_t^{T_1} = 0 = \rho$ and $\eta_t^{P_2} = \eta_t^P$, $\eta_t^{T_2} = \eta_t^T$. The bivariate moving average of the price vector $X_t = (F_t\ S_t)'$ in (1) then becomes:

$$\begin{pmatrix} \triangle F_t \\ \triangle S_t \end{pmatrix} = \Psi(L) \begin{pmatrix} e_t^f \\ e_t^s \end{pmatrix}, \tag{57}$$

Figure 3: **Time Synchronism**



where $e_t = (e_{f,t}\ e_{s,t})'$ is a 2×1 vector of innovations satisfying $\mathrm{E}(e_t) = 0$ and

$$\mathrm{E}[e_t e_s'] = \begin{cases} 0 & \text{if } t \neq s, \\ \Omega & \text{otherwise.} \end{cases} \tag{58}$$

So, the P-T decomposition's task is to find a matrix $D_0 = \begin{pmatrix} d_1 & d_2 \\ d_3 & d_4 \end{pmatrix}$ such that the following three conditions, which correspond to the three conditions in Subsection 2.3, are met:

(i) P-T decomposition:

$$e_t = D_0\, \eta_t = D_0 \begin{pmatrix} \eta_t^P \\ \eta_t^T \end{pmatrix} = \begin{pmatrix} d_1 & d_2 \\ d_3 & d_4 \end{pmatrix} \begin{pmatrix} \eta_t^P \\ \eta_t^T \end{pmatrix}; \tag{59}$$

(ii) From defining characteristics of the P-T innovations:

$$\psi_1 d_1 + \psi_2 d_3 = 1, \tag{60}$$

$$\frac{d_2}{d_4} = -\frac{\psi_2}{\psi_1}. \tag{61}$$

(iii) Orthogonality condition (by construction)

$$\mathrm{cov}(\eta_t^P, \eta_t^T) = 0. \tag{62}$$

Repeating the solution in the asynchronism case, we have the following synchrony solution to d_1, d_2, d_3, d_4 problem:

$$\begin{aligned} d_2 &= -\kappa\psi_2, && (63)\\ d_4 &= \kappa\psi_1, && (64)\\ d_1 &= \frac{\psi_1 \mathrm{var}(e_t^f) + \psi_2 \mathrm{cov}(e_t^f, e_t^s)}{\psi_1^2 \mathrm{var}(e_t^f) + 2\psi_1\psi_2 \mathrm{cov}(e_t^f, e_t^s) + \psi_2^2 \mathrm{var}(e_t^s)}, && (65)\\ d_3 &= \frac{\psi_1 \mathrm{cov}(e_t^f, e_t^s) + \psi_2 \mathrm{var}(e_t^s)}{\psi_1^2 \mathrm{var}(e_t^f) + 2\psi_1\psi_2 \mathrm{cov}(e_t^f, e_t^s) + \psi_2^2 \mathrm{var}(e_t^s)}. && (66) \end{aligned}$$

where the parameter $det(D_0) = \kappa$ can be any real number as one d can be fixed arbitrarily.

With this solution, we can write the P-T innovations for the synchronous case as:

$$\begin{aligned} \begin{pmatrix} \eta_t^P \\ \eta_t^T \end{pmatrix} &= D_0^{-1} e_t, \\ &= \frac{1}{\det(D_0)} \begin{pmatrix} d_4 & -d_2 \\ -d_3 & d_1 \end{pmatrix} \begin{pmatrix} e_t^f \\ e_t^s \end{pmatrix}, \\ &= \begin{pmatrix} \psi_1 e_t^f + \psi_2 e_t^s \\ \frac{-d_3}{\kappa} e_t^f + \frac{d_1}{\kappa} e_t^s \end{pmatrix}. && (67) \end{aligned}$$

We lastly link this to the approach by Gonzalo & Ng (2001). In their procedure, the VECM residuals are orthogonalized in two steps. The first step separates the permanent from the transitory shocks: two numbers $u_t^P = \gamma_\perp' e_t$[7] and $u_t^T = \alpha' e_t$ are chosen to be the permanent and transitory shocks, respectively. The second step uses a Choleski decomposition to obtain a set of permanent and transitory shocks, $\eta_t = (\eta_t^P \ \eta_t^T)'$, that are mutually orthogonal: $\eta_t = H^{-1} u_t$ where $u_t = (u_t^P \ u_t^T)'$ and H is the Choleski decomposition of $\mathrm{cov}(u)$. According to Gonzalo and Ng, the choice of P-T shocks in the first step is motivated by the reason that

> "$\gamma_\perp'$ and α' will 'knock out' the appropriate terms in the VECM and moving-average representations of $\triangle X_t$ to isolate those components with the desired degree of integration".

However, in our solution for the P-T decomposition, the choice for the transitory shocks does not require any cointegrating vector α, which means that their choice $u_t^T = \alpha' e_t$ is arbitrary. Except for this difference, our solution to the task of decomposing permanent and transitory

[7] $\gamma_\perp$ is the 2x1 orthogonal complement of γ such that $\gamma' \gamma_\perp = 0$. According to Baillie. et. al. (2002), $\gamma_\perp' = (\psi_1 \ \psi_2)'$

innovations from the VECM residuals is similar to their two-step procedure, at least in the case of synchronism.

4 Empirical Results

The unit-root test was performed for all the 71 stocks. Then, the cointegrated model VEC($K-1$) in (5) with K chosen to minimize the Bayesian Information Criterion (BIC) was estimated for all of them. For the sake of brevity, we omit the full results and just describe the general characteristics of the coefficient estimates. For all the stocks, the prices are consistent with the hypotheses of a unit root and of a cointegration relation between the forward and spot prices. In addition, the coefficients of the cointegration vector α and adjustment speed γ are clearly different from zero, and the cointegration vector $\alpha^{'}$ is not significantly different from the vector $(1\ \ -1)^{'}$. This is, of course, consistent with the prediction that the difference between the two prices is stationary.

The correlation between $e_{f,t}$ and $e_{s,t+\triangle}$ is significant, as expected since the two 24-hour periods overlap by some 20.5 hours; more surprisingly, though, the cross-correlation is not. Notably, for almost all the cases (70 out of 71 stocks), the correlation between the forward residual, $e_{f,t}$, and the lagged spot residual, $e_{s,t+\triangle-1}$, is not statistically significant even though the two open-to-open periods overlap by about 3.5 hours. As a result, the ratio of the permanent and transitory shocks' volatility ρ estimated in (11) is insignificant. Therefore, although the opening forward and spot prices are determined at different moments on a day, the volatility of the permanent and transitory innovations during this period of $\triangle = 3.5$ hours are insignificant (see formula (8) for reference). One implication, with hindsight, is that we could have ignored the asynchrony and just followed the standard procedure presented in subsection 3.5. We come back to the economic interpretation of this finding in the final section.

The matrix polynomial $\Psi(L)$ of the VMA presentation (1) is then estimated from the VEC($K-1$) using the procedure in Yan & Zivot (2007). For each stock, the price-adjustment speed, k_f and k_s, and price-discovery-efficiency losses $PDEL_2^F(K^*)$ and $PDEL_2^S(K^*)$ in (55) and (56), are then estimated. We do not present results on the price-discovery-efficiency losses $PDEL_1^F(K^*)$ and $PDEL_1^S(K^*)$ that correspond to $\eta_t^{P_1}$ because the volatility of $\eta_t^{P_1}$ is insignificant as shown above. In addition, the 5- and 95% quantile confidence intervals of the price discovery measure are computed using the bootstrap procedure with 1000 sampling times. The next subsections present the empirical results in detail.

4.1 Speed of Price Adjustment

Figure 4: **Speed of Price Adjustment, forward v spot**

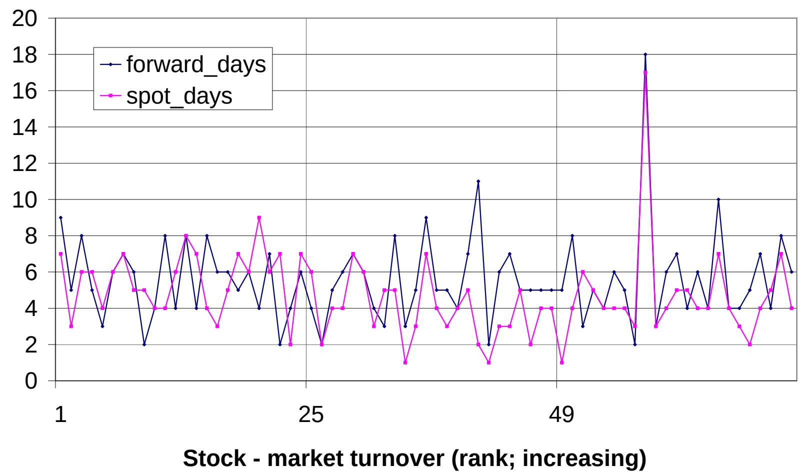


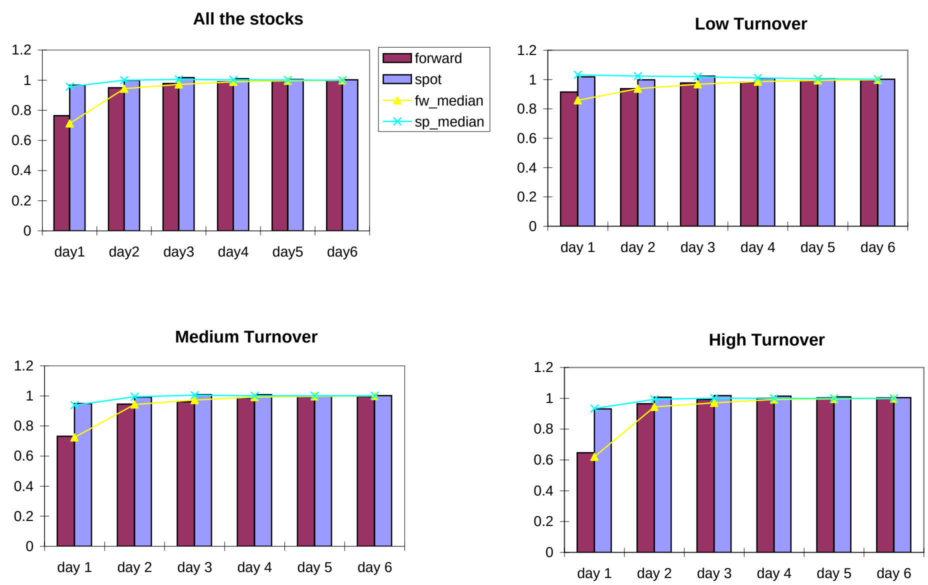
Table 3: **Speed of Price Adjustment**

	Forward mean*	Forward median*	Spot mean*	Spot median*	forward adjusts slower**	equal speed**	forward adjusts faster**
All	5.58	5	4.73	4	41	15	15
Low	5.54	6	5.58	6	10	5	9
Medium	5.39	5	3.87	4	16	5	2
High	5.79	5	4.71	4	15	5	4

* in number of days;
** in number of cases.

Figure 4 and Table 3 report the speed of price adjustment of the forward and spot markets, which is measured by the number of days the market prices take to fully incorporate a one-unit change in the fundamental innovation. The stocks are ranked on the basis of their average daily turnover, from low to high. Out of the total 71 stocks, forward prices adjust more slowly in 41 cases (10 low-turnover, 16 medium, and 15 high), and faster in 15 cases (9 low-turnover, 2 medium, 4 high). In the remaining 15 cases (5 low-turnover, 5 medium, 5 high), the forward and spot prices have an equal speed. In addition, Table 3 also presents the mean and median of the price adjustment speed of the two markets. The general picture from these summary statistics is that on average it takes around 5.5 days for the forward market to fully response to a one-unit shock, whereas it takes around 4.5 days for the spot market to do so. Put differently,

Figure 5: **Mean and Median of Price Adjustment**



on average the forward market is one day slower than the spot one in responding to the shock.

Figure 5 and Table 4 report the mean and median of the price response for all the stocks and for each turnover group in the first six days after a one-unit shock. On the first day, on average of the total 71 stocks, the forward market incorporates 76% of the shock (91% low-turnover, 73% medium, and 65% high), whereas for the spot one the figure is 97% (102% low-turnover, 95% medium, and 93% high). On the second day, the spot prices almost hit the new fundamental value, while the forward prices are still around 5% below. By the fifth and sixth days, both markets have responded fully to the shock. Another characteristic being seen from Figure 5 and Table 4 is that the spot prices seem to overreact to the shock on the third and the fourth days before going back to the new fundamental level. Consequently, one question arising from this feature is that whether they accumulate more error during the process of fully impounding this new information. The next part will look into this issue.

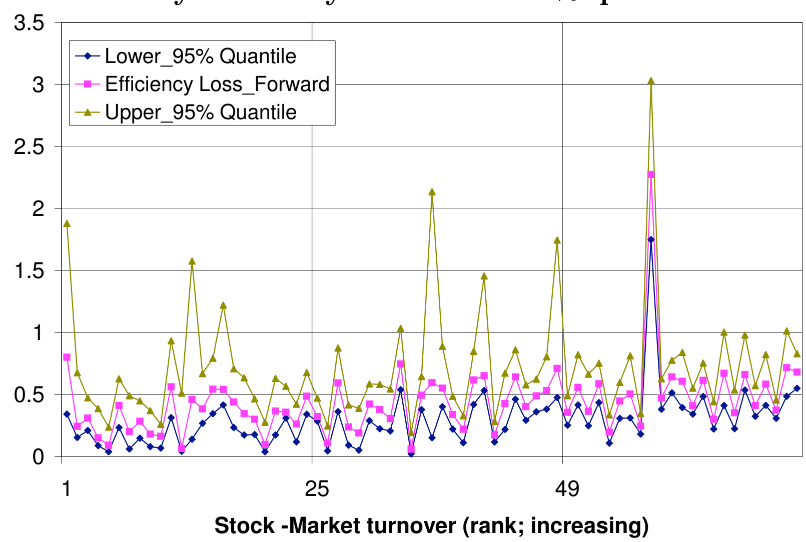
In sum, the forward tier is slower than the spot one in reacting to new fundamental information. This comparative slowness in the forward market is unexpected, in view of its higher volume, superior organization, and lower costs. There also seems to be no clear tendency for the adjustment period to shorten in the forward market, the higher the stock's liquidity.

Table 4: **Price Adjustment, Mean & Median**

		day 1 mean	day 1 median	day 2 mean	day 2 median	day 3 mean	day 3 median	day 4 mean	day 4 median	day 5 mean	day 5 median	day 6 mean	day 6 median
All	fw	**0.76**	0.71	**0.95**	0.94	**0.98**	0.97	**0.99**	0.99	**1.00**	1.00	**1.00**	1.00
	sp	**0.97**	0.96	**1.00**	1.00	**1.02**	1.00	**1.01**	1.00	**1.00**	1.00	**1.00**	1.00
Low	fw	**0.91**	0.86	**0.94**	0.94	**0.98**	0.97	**0.98**	0.99	**0.99**	0.99	**1.00**	1.00
	sp	**1.02**	1.03	**1.00**	1.02	**1.02**	1.02	**1.01**	1.01	**1.01**	1.01	**1.00**	1.00
Medium	fw	**0.73**	0.72	**0.95**	0.94	**0.96**	0.97	**0.99**	0.99	**1.00**	1.00	**1.00**	1.00
	sp	**0.95**	0.94	**0.99**	1.00	**1.01**	1.00	**1.01**	1.00	**1.00**	1.00	**1.00**	1.00
High	fw	**0.65**	0.62	**0.96**	0.94	**0.99**	0.97	**1.00**	0.99	**1.00**	1.00	**1.00**	1.00
	sp	**0.93**	0.93	**1.01**	0.99	**1.02**	1.00	**1.01**	1.00	**1.01**	1.00	**1.00**	1.00

Key The entries 'low', 'medium', and 'high' refer to turnover classes, each containing 24, 23 and 24 stocks, respectively; 'fw' and 'sp' are for forward and spot, respectively.

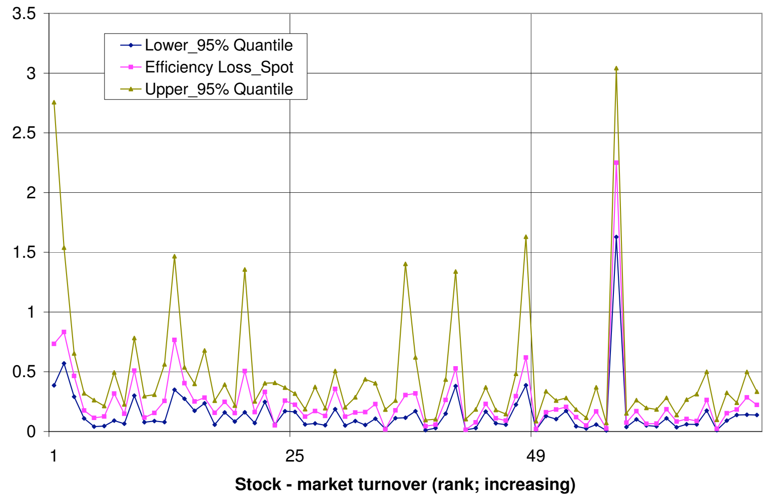
Figure 6: **Price Discovery Efficiency Loss and 5-95% quantile intervals, Forward**



4.2 Price Discovery Efficiency Loss

Recall that the efficiency loss is defined on the basis of the estimated sensitivities of the expected prices for days t, $t+1$, ... , $t+K^*$ to a one-unit shock η^{P_i} occuring at time t. In a perfect market, this sensitivity equals unity for the contemporaneous price. So Yan and Zivot (2007) add up all absolute deviations between the sensitivities and unity, over the number of days it takes for the cumulative adjustment to become complete. Figure 6 presents the point estimates and the 5- and 95% quantile intervals of price-discovery-efficiency-loss in the forward market, Figure 7 for the spot tier, and Figure 8 for the difference of the loss between the two markets. Since the statistic is a sum of nonnegative numbers, its distribution is highly right-skewed. In the forward market, there seems to be a (weak) tendency for the losses to rise when turnover is

Figure 7: **Price Discovery Efficiency Loss and 5-95% quantile intervals, Spot**



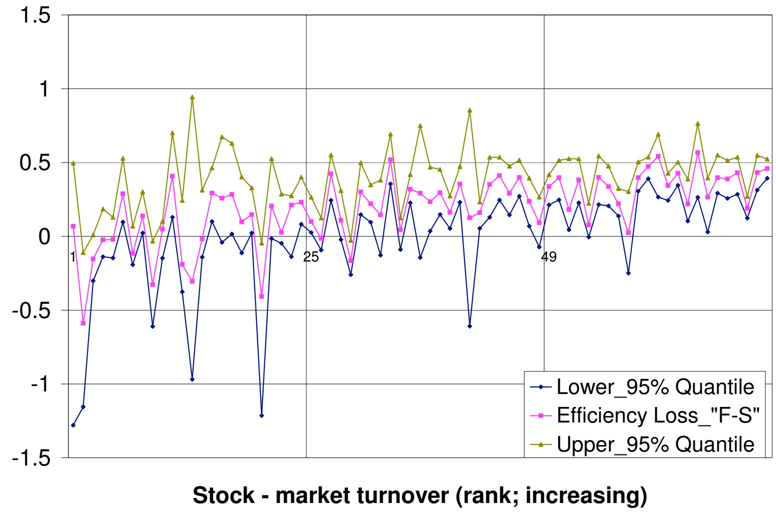
higher, which is a puzzling effect; for the spot tier there seems to be no such tendency. To get a clearer picture we show the differences, stock by stock, of the efficiency losses forward minus spot, along with the 5-95% confidence intervals from the bootstrap. Out of the 71 total number of cases, the forward prices have a higher efficiency loss in 59 cases, of which 41 are significant with the lower confidence bound exceeding zero. So, all the empirical results again suggest that the spot market contributed more to the price discovery process than the forward one.

Equally unexplicable, the bad performance of the forward market is worst among the most active stocks: for each and every stock in the top third the forward market seems slower, and likewise for all but two of the middle third; only in the low-activity stocks the performance is about even. The significant ones are also concentrated: 21/23 in the top, 17/24 in the middle, and 7/24 in the lower third. Of the twelve cases where the forward market does better, only four are significant, of which three are in the low-activity third.

5 Conclusions

In this paper, we propose an adjustment to the standard structural cointegration model for price changes in arbitrage linked markets. In general, this adjusted model allows us to cope with asynchronism problem between the spot and forward markets when examining the prices' dynamic process of converging to the new equilibrium prices. Additionally, it helps to reveal whether the asynchronism in the observed prices is significant or not.

Figure 8: **Price Discovery Efficiency Loss - Forward-Spot (5-95% quantile intervals)**



Upon fitting the adjusted model with the data in the BSE, we find that the asynchronism problem is statistically insignificant for almost all the stocks. In the math of this model, the interpretation would be that there is no news whatsoever between 10:00 and 13:30, but this is of course unconvincing as a description of reality. A more reasonable explanation for this could be that any fundamental shocks happening during the morning were not reflected in the opening spot price observed at 1.30 p.m. of the same day. For example, the bulk of the spot limit orders may have been submitted at the same time as forward ones and may rarely have been updated in light of what happened in the forward tier. This would fit in with the view of one practitioner, who described the spot tier as the market for smaller private investors who read newspapers after work and submit orders before work the next day.

Yet that view, if true, does not seem to mean that these small investors are less adept at digesting the news. With the adjusted structural cointegration model, we perform an impulse response analysis to examine the price adjustment process of the forward and spot markets following a one-unit change in the permanent innovation, i.e after the permanent innovation increases by BEF 1. The result shows that after this event it takes longer time for the forward prices to incorporate the full one-unit shock in the majority of cases. Next, we apply the measure of the price-discovery-dynamics proposed by Yan and Zivot (2007) to estimate the accumulated error that each market makes while moving to the new fundamental value following a fundamental shock. Our results are that the forward market makes significantly more errors than the spot one. So, we can conclude that the forward market failed to play the superior role one would have expected. There seems to be no obvious reason why this would be the case. Equally unexplicably, the comparatively poor performance is concentrated among the bigger stocks.

References

Baillie, R.T., Booth, G.G., Tse, Y., and Tatyana, Z., 2002, Price discovery and common factor model, Journal of Financial Markets 5, 309-321.

Bui, T. N. T., Sercu, P. (2007), Segmentation and price discovery in the two-tier Brussels Stock Exchange, Working paper, Department of Accountancy, Finance & Insurance, K.U.L, July 2007.

De Jong, F., 2002, Measures of contributions to price discovery: a comparison, Journal of Financial Markets 5, 323-327.

Garbade, K.D. and Silber, W.L. (1983), Price movements and price discovery in futures and cash markets, *Review of Economics and Statistics* 65(2), 289-287.

Gonzalo, J. and Granger, C. , Estimation of common long-memory components in cointegrated systems. Journal of Business and Economic Statistics 13 (1995), pp. 19.

Gonzalo, J., Ng, S. (2001), A systematic framework for analyzing the dynamic effects of permanent and transitory shocks, *Journal of Economic Dynamics* & *Control* 25, 1527-1546.

Harris, F.H.D., McInish, T.H., Wood, R.A., 2002. Common factor components vs. information shares: a reply. Journal of Financial Markets 5, 341-348 .

Hasbouck, J. (1995), One security, many markets: Determining the contributions to price discovery, *Journal of Finance* 4, September 1995.

Hasbrouck, J., 2002. Stalking the efficient price in market microstructure specifications: an overview. Journal of Financial Markets 5, 329-339.

Lehmann, B.N., 2002. Some desiderata for the measurement of price discovery across markets. Journal of Financial Markets 5, 259-276.

Schreiber, Paul S., and Robert A. Schwartz, 1986, Price discovery in securities markets, *Journal of Portfolio Management* 12, 43-48.

Yan, B., Zivot, E. (2007), The dynamics of price discovery, Working paper, Department of Economics, University of Washington, February 2007.

Appendix A

This appendix shows that (49) and (50) are the unique solution of the four equations (34), (45), (46), and (47), which are rewritten below:

$$\begin{aligned}
\psi_1 d_1 + \psi_2 d_3 &= 1 && (68)\\
\mathrm{var}(e_{f,t}) &= d_1^2(1+\rho)\mathrm{var}(\eta_t^{P_2}) + d_2^2(1+\rho)\mathrm{var}(\eta_t^{T_2}), && (69)\\
\mathrm{var}(e_{s,t+\triangle}) &= d_3^2(1+\rho)\mathrm{var}(\eta_t^{P_2}) + d_4^2(1+\rho)\mathrm{var}(\eta_t^{T_2}) && (70)\\
\mathrm{cov}(e_{f,t}, e_{s,t+\triangle}) &= d_1 d_3 \mathrm{var}(\eta_t^{P_2}) + d_2 d_4 \mathrm{var}(\eta_t^{T_2}) && (71)
\end{aligned}$$

where d_2, d_4, and ρ are coefficients that can be estimated as $d_2 = -\psi_2$, $d_4 = \psi_1$, and $\rho = \frac{\mathrm{cov}(e_{f,t}, e_{s,t+\triangle-1})}{\mathrm{cov}(e_{f,t}, e_{s,t+\triangle})}$. From (69) and (70) we have

$$\begin{pmatrix} \mathrm{var}(e_{f,t}) \\ \mathrm{var}(e_{s,t+\triangle}) \end{pmatrix} = (1+\rho)\begin{pmatrix} d_1^2 & d_2^2 \\ d_3^2 & d_4^2 \end{pmatrix}\begin{pmatrix} \mathrm{var}(\eta_t^{P_2}) \\ \mathrm{var}(\eta_t^{T_2}) \end{pmatrix}. \tag{72}$$

or

$$\begin{aligned}
\begin{pmatrix} \mathrm{var}(\eta_t^{P_2}) \\ \mathrm{var}(\eta_t^{T_2}) \end{pmatrix} &= \frac{1}{1+\rho}\begin{pmatrix} d_1^2 & d_2^2 \\ d_3^2 & d_4^2 \end{pmatrix}^{-1}\begin{pmatrix} \mathrm{var}(e_{f,t}) \\ \mathrm{var}(e_{s,t+\triangle}) \end{pmatrix}, \\
&= \left(\frac{1}{1+\rho}\right)\left(\frac{1}{d_1^2 d_4^2 - d_2^2 d_3^2}\right)\begin{pmatrix} d_4^2 & -d_2^2 \\ -d_3^2 & d_1^2 \end{pmatrix}\begin{pmatrix} \mathrm{var}(e_{f,t}) \\ \mathrm{var}(e_{s,t+\triangle}) \end{pmatrix}. && (73)
\end{aligned}$$

Replace (73) into (71) we have

$$\mathrm{cov}(e_{f,t}, e_{s,t+\triangle}) = \left(\frac{1}{1+\rho}\right)\left(\frac{1}{d_1^2 d_4^2 - d_2^2 d_3^2}\right)\begin{pmatrix} d_1 d_3 & d_2 d_4 \end{pmatrix}\begin{pmatrix} d_4^2 & -d_2^2 \\ -d_3^2 & d_1^2 \end{pmatrix}\begin{pmatrix} \mathrm{var}(e_{f,t}) \\ \mathrm{var}(e_{s,t+\triangle}) \end{pmatrix} \tag{74}$$

Replace d_2 by $(-d_4\frac{\psi_2}{\psi_1})$, we have:

$$\begin{aligned}
d_1^2 d_4^2 - d_2^2 d_3^2 &= d_1^2 d_4^2 - \left(\frac{\psi_2^2}{\psi_1^2} d_4^2\right) d_3^2 \\
&= \frac{d_4^2}{\psi_1^2}(d_1\psi_1 - \psi_2 d_3)(d_1\psi_1 + \psi_2 d_3) \\
&= \frac{d_4^2}{\psi_1^2}(d_1\psi_1 - \psi_2 d_3) && (75)
\end{aligned}$$

$$\begin{aligned}
\begin{pmatrix} d_1 d_3 & d_2 d_4 \end{pmatrix}\begin{pmatrix} d_4^2 & -d_2^2 \\ -d_3^2 & d_1^2 \end{pmatrix} &= \begin{pmatrix} d_1 d_3 d_4^2 - d_2 d_4 d_3^2 & d_2 d_4 d_1^2 - d_1 d_3 d_2^2 \end{pmatrix} \\
&= \begin{pmatrix} d_1 d_3 d_4^2 - (-\frac{\psi_2}{\psi_1} d_4) d_4 d_3^2 & d_1 d_3 d_4^2 - (-\frac{\psi_2}{\psi_1} d_4) d_4 d_3^2 \end{pmatrix} \\
&= \begin{pmatrix} d_4^2 \frac{d_3}{\psi_1}(\psi_1 d_1 + \psi_2 d_3) & -d_4^2 \frac{\psi_2}{\psi_1^2} d_1(\psi_2 d_3 + \psi_1 d_1) \end{pmatrix} \\
&= \begin{pmatrix} d_4^2 \frac{d_3}{\psi_1} & (-d_4^2 d_1 \frac{\psi_2}{\psi_1^2}) \end{pmatrix} \\
&= \frac{d_4^2}{\psi_1^2}\begin{pmatrix} \psi_1 d_3 & (\psi_2 d_1) \end{pmatrix} && (76)
\end{aligned}$$

Replace (75)and (76) into (74) we have:

$$\begin{aligned}
\mathrm{cov}(e_{f,t}, e_{s,t+\triangle}) &= (\frac{1}{1+\rho})(\frac{1}{\frac{d_4^2}{\psi_1^2}(d_1\psi_1 - \psi_2 d_3)})(\frac{d_4^2}{\psi_1^2})\left(\begin{array}{cc}\psi_1 d_3 & (-\psi_2 d_1)\end{array}\right)\left(\begin{array}{c}\mathrm{var}(e_{f,t})\\ \mathrm{var}(e_{s,t+\triangle})\end{array}\right)\\
&= (\frac{1}{1+\rho})(\frac{1}{d_1\psi_1 - \psi_2 d_3})\left(\begin{array}{cc}\psi_1 d_3 & (-\psi_2 d_1)\end{array}\right)\left(\begin{array}{c}\mathrm{var}(e_{f,t})\\ \mathrm{var}(e_{s,t+\triangle})\end{array}\right) \qquad (77)
\end{aligned}$$

From (68) and (77), we have the solution for d_1 and d_3:

$$d_1 = \frac{(1+\rho)\psi_2 \mathrm{cov}(e_{f,t}, e_{s,t+\triangle}) + \psi_1 \mathrm{var}(e_{f,t})}{\psi_1^2 \mathrm{var}(e_{f,t}) + 2(1+\rho)\psi_1\psi_2 \mathrm{cov}(e_{f,t}, e_{s,t+\triangle}) + \psi_2^2 \mathrm{var}(e_{s,t+\triangle})}, \qquad (78)$$

$$d_3 = \frac{(1+\rho)\psi_1 \mathrm{cov}(e_{f,t}, e_{s,t+\triangle}) + \psi_2 \mathrm{var}(e_{s,t+\triangle})}{\psi_1^2 \mathrm{var}(e_{f,t}) + 2(1+\rho)\psi_1\psi_2 \mathrm{cov}(e_{f,t}, e_{s,t+\triangle}) + \psi_2^2 \mathrm{var}(e_{s,t+\triangle})}. \qquad (79)$$

Table 5: **Speed of Price Adjustment**

Low-turnover stocks			Medium-turnover stocks			High-turnover stocks		
stockid	days, forward	days, spot	stockid	days, forward	days, spot	stockid	days, forward	days, spot
20	9	7	15	4	6	25	5	4
82	5	3	287	2	2	40	5	1
11	8	6	268	5	4	57	8	4
21	5	6	52	6	4	68	3	6
269	3	4	9	7	7	35	5	5
24	6	6	36	6	6	34	4	4
56	7	7	260	4	3	30	6	4
8	6	5	66	3	5	13	5	4
23	2	5	76	8	5	39	2	3
293	4	4	290	3	1	61	18	17
7	8	4	71	5	3	67	3	3
32	4	6	10	9	7	46	6	4
77	8	8	72	5	4	78	7	5
55	4	7	29	5	3	19	4	5
70	8	4	60	4	4	1	6	4
16	6	3	289	7	5	65	4	4
31	6	5	5	11	2	59	10	7
73	5	7	48	2	1	47	4	4
291	6	6	26	6	3	43	4	3
51	4	9	58	7	3	69	5	2
296	7	6	64	5	5	27	7	4
53	2	7	37	5	2	41	4	5
62	4	2	17	5	4	63	8	7
292	6	7				42	6	4

Table 6: **Price Discovery Efficiency Loss - Forward (5-95% quantile intervals)**

Low-turnover stocks				Medium-turnover stocks				High-turnover stocks			
stockid	5%	EffLoss	95%	stockid	5%	EffLoss	95%	stockid	5%	EffLoss	95%
20	0.34	0.80	1.88	15	0.29	0.32	0.47	25	0.48	0.71	1.75
82	0.16	0.24	0.68	287	0.05	0.11	0.25	40	0.25	0.36	0.49
11	0.21	0.31	0.48	268	0.36	0.60	0.88	57	0.42	0.56	0.82
21	0.09	0.15	0.39	52	0.09	0.24	0.42	68	0.25	0.37	0.67
269	0.04	0.09	0.24	9	0.05	0.19	0.39	35	0.44	0.59	0.75
24	0.23	0.41	0.63	36	0.29	0.42	0.59	34	0.11	0.20	0.34
56	0.06	0.20	0.49	260	0.23	0.38	0.58	30	0.31	0.45	0.60
8	0.15	0.29	0.45	66	0.21	0.31	0.55	13	0.31	0.51	0.81
23	0.08	0.18	0.37	76	0.54	0.75	1.04	39	0.18	0.25	0.35
293	0.07	0.17	0.26	290	0.02	0.06	0.19	61	1.75	2.27	3.03
7	0.32	0.56	0.93	71	0.38	0.50	0.65	67	0.38	0.47	0.63
32	0.04	0.07	0.51	10	0.15	0.60	2.14	46	0.52	0.64	0.78
77	0.14	0.46	1.58	72	0.40	0.55	0.89	78	0.40	0.61	0.84
55	0.27	0.39	0.67	29	0.22	0.34	0.49	19	0.34	0.41	0.55
70	0.35	0.54	0.79	60	0.11	0.22	0.33	1	0.49	0.61	0.76
16	0.42	0.54	1.22	289	0.42	0.62	0.85	65	0.22	0.30	0.44
31	0.24	0.44	0.71	5	0.53	0.65	1.46	59	0.41	0.67	1.00
73	0.17	0.35	0.64	48	0.12	0.18	0.28	47	0.23	0.35	0.54
291	0.18	0.30	0.47	26	0.22	0.43	0.67	43	0.54	0.66	0.98
51	0.04	0.10	0.28	58	0.46	0.64	0.86	69	0.33	0.41	0.57
296	0.17	0.37	0.63	64	0.29	0.40	0.58	27	0.41	0.58	0.82
53	0.31	0.36	0.57	37	0.36	0.49	0.63	41	0.31	0.37	0.46
62	0.12	0.26	0.42	17	0.38	0.53	0.81	63	0.49	0.72	1.01
292	0.34	0.49	0.68					42	0.55	0.68	0.83

Table 7: **Price Discovery Efficiency Loss - Spot (5-95% quantile intervals)**

Low-turnover stocks				Medium-turnover stocks				High-turnover stocks			
stockid	5%	EffLoss	95%	stockid	5%	EffLoss	95%	stockid	5%	EffLoss	95%
20	0.39	0.73	2.76	15	0.16	0.22	0.32	25	0.39	0.62	1.63
82	0.57	0.83	1.54	287	0.06	0.12	0.19	40	0.01	0.02	0.09
11	0.29	0.47	0.65	268	0.07	0.17	0.37	57	0.13	0.16	0.34
21	0.11	0.18	0.32	52	0.05	0.13	0.20	68	0.10	0.18	0.26
269	0.04	0.11	0.26	9	0.19	0.36	0.51	35	0.17	0.21	0.28
24	0.05	0.13	0.21	36	0.05	0.12	0.20	34	0.04	0.12	0.18
56	0.09	0.32	0.49	260	0.09	0.16	0.29	30	0.03	0.05	0.12
8	0.07	0.15	0.23	66	0.06	0.16	0.44	13	0.06	0.17	0.37
23	0.30	0.51	0.78	76	0.11	0.23	0.40	39	0.01	0.03	0.07
293	0.08	0.12	0.30	290	0.02	0.02	0.18	61	1.63	2.25	3.04
7	0.09	0.15	0.31	71	0.11	0.18	0.26	67	0.04	0.07	0.15
32	0.08	0.26	0.56	10	0.12	0.30	1.40	46	0.10	0.17	0.26
77	0.35	0.77	1.47	72	0.17	0.32	0.62	78	0.05	0.07	0.20
55	0.27	0.40	0.54	29	0.01	0.04	0.10	19	0.04	0.07	0.19
70	0.17	0.25	0.40	60	0.03	0.06	0.10	1	0.11	0.19	0.28
16	0.24	0.28	0.68	289	0.15	0.26	0.44	65	0.04	0.08	0.14
31	0.06	0.16	0.26	5	0.38	0.53	1.34	59	0.06	0.11	0.27
73	0.16	0.25	0.39	48	0.01	0.02	0.10	47	0.06	0.09	0.31
291	0.08	0.15	0.22	26	0.03	0.08	0.19	43	0.18	0.26	0.50
51	0.16	0.51	1.36	58	0.17	0.23	0.37	69	0.01	0.02	0.10
296	0.07	0.16	0.25	64	0.07	0.11	0.18	27	0.09	0.15	0.32
53	0.25	0.33	0.40	37	0.06	0.09	0.15	41	0.14	0.18	0.24
62	0.06	0.05	0.41	17	0.23	0.30	0.48	63	0.14	0.29	0.50
292	0.17	0.26	0.37					42	0.14	0.22	0.33

Table 8: **Price Discovery Efficiency Loss - Forward-Spot (5-95% quantile intervals)**

Low-turnover stocks				Medium-turnover stocks				High-turnover stocks			
stockid	5%	EffLoss	95%	stockid	5%	EffLoss	95%	stockid	5%	EffLoss	95%
20	-1.28	0.07	0.50	15	0.03	0.10	0.26	25	-0.07	0.09	0.27
82	-1.16	-0.59	-0.11	287	-0.09	-0.01	0.12	40	0.21	0.34	0.42
11	-0.30	-0.15	0.01	268	0.24	0.42	0.55	57	0.25	0.40	0.52
21	-0.14	-0.02	0.19	52	-0.02	0.11	0.31	68	0.04	0.18	0.53
269	-0.15	-0.02	0.13	9	-0.26	-0.17	-0.03	35	0.23	0.38	0.52
24	0.10	0.29	0.53	36	0.15	0.30	0.50	34	0.00	0.08	0.22
56	-0.19	-0.12	0.07	260	0.10	0.22	0.35	30	0.22	0.40	0.55
8	0.02	0.14	0.30	66	-0.13	0.15	0.38	13	0.21	0.34	0.48
23	-0.61	-0.33	-0.03	76	0.36	0.52	0.69	39	0.14	0.22	0.32
293	-0.15	0.05	0.10	290	-0.09	0.04	0.12	61	-0.25	0.02	0.30
7	0.13	0.41	0.70	71	0.23	0.32	0.42	67	0.31	0.40	0.50
32	-0.38	-0.19	0.24	10	-0.14	0.29	0.75	46	0.39	0.47	0.54
77	-0.97	-0.31	0.94	72	0.04	0.23	0.47	78	0.27	0.54	0.69
55	-0.14	-0.02	0.31	29	0.15	0.30	0.45	19	0.24	0.34	0.43
70	0.10	0.29	0.46	60	0.05	0.16	0.27	1	0.35	0.43	0.50
16	-0.04	0.26	0.67	289	0.23	0.35	0.47	65	0.10	0.22	0.39
31	0.02	0.28	0.63	5	-0.61	0.13	0.85	59	0.26	0.57	0.76
73	-0.11	0.10	0.40	48	0.05	0.16	0.23	47	0.03	0.27	0.40
291	0.02	0.15	0.33	26	0.13	0.35	0.54	43	0.29	0.40	0.55
51	-1.21	-0.41	-0.05	58	0.25	0.41	0.54	69	0.26	0.39	0.51
296	-0.02	0.21	0.53	64	0.15	0.29	0.48	27	0.29	0.43	0.54
53	-0.05	0.03	0.29	37	0.27	0.40	0.52	41	0.12	0.19	0.27
62	-0.14	0.21	0.27	17	0.07	0.24	0.39	63	0.31	0.43	0.55
292	0.08	0.23	0.40					42	0.39	0.46	0.52